Discrete Mathematics

Grades 9-12 **1 credit**

Prerequisite: Precalculus G/T

This course is an introduction to the study of Discrete Mathematics, a branch of contemporary mathematics that develops reasoning and problem-solving abilities, with an emphasis on proof. Topics include Logic, Mathematical Reasoning and Proof, Set Theory, Combinatorics, Probability, Cryptology, and Graph Theory. Course requirements are rigorous, with an emphasis on mathematical reasoning and communication. This course is intended for students capable of and interested in progressing through the concepts of discrete mathematics in more depth and at an accelerated rate. Graphing calculators are an integral part of this course.

Discrete Mathematics
Essential Curriculum

UNIT I: Logic

Goal. The student will demonstrate the ability to use mathematical logic to solve problems.

Objectives – The student will be able to:

a. represent English-language statements using symbolic logic notation.
b. use and interpret relational conjunctions (*and, or, xor, not*), terms of causation (*if… then*) and equivalence (*if and only if*).
c. use truth tables to analyze the truth values of compound statements based on the truth values of their components.
d. use truth tables to determine if two statements are logically equivalent.
e. use truth tables to identify tautologies and contradictions.
f. identify the hypothesis and conclusion of a conditional statement.
g. interpret English-language statements (*if, only if, necessary, sufficient*) by converting each to the standard *"if… then"* form.
h. write the converse, inverse, and contrapositive of a conditional statement.
i. describe a counterexample for a conditional statement.
j. write the negation of a conditional statement.
k. write the negation of statements with quantifiers *"for all"* and *"there exists"* and write the negation of statements with multiple quantifiers.
l. recognize *modus ponens* and *modus tollens* as valid argument structures.
m. recognize the *converse fallacy* and *inverse fallacy* as invalid argument structures.
n. use truth tables to determine if a given argument structure is valid.

UNIT II: Mathematical Reasoning and Proof

Goal. The student will demonstrate the ability to use iterative and recursive processes to prove properties of integers.

Objectives – The student will be able to:

a. write a direct proof for a simple implication involving basic properties of integers (like "*even*" and "*odd*").

b. prove an implication by forming its contrapositive, then proving that contrapositive implication.
c. write a proof for a statement whose hypothesis or conclusion states that a particular integer is divisible by another.
d. use the division theorem to define cases for a proof.
e. write a proof for a statement whose hypothesis or conclusion states that a particular real number is rational.
f. use the principle of mathematical induction to prove the terms of a sequence (whose recursive description is given) satisfy a given closed formula.
g. use the principle of mathematical induction to prove a summation satisfies a given closed formula.
h. use the principle of mathematical induction to establish divisibility properties.
i. write a proof by contradiction for a statement about basic properties of numbers.
j. recognize various forms of proof (direct proof, proof by contrapositive, proof by cases, proof by exhaustion, proof by mathematical induction, proof by contradiction, existence proof).

UNIT III: Set Theory

Goal. The student will demonstrate the ability to use sets to codify mathematical objects.

Objectives – The student will be able to:

a. represent a set using set-builder notation.
b. give examples of finite and infinite sets.
c. build new sets from existing sets using various combinations of the set operations *intersection union, difference*, and *complement*.
d. determine whether two sets are equal by determining whether each is a subset of the other.
e. use Venn diagrams to illustrate and investigate properties of set operations.
f. apply the Inclusion-Exclusion Principle, for two and three sets, to determine the size of the union of sets.
g. calculate and describe the Cartesian product of *n* sets.
h. find the partition of a set satisfying given conditions.
i. calculate and describe the power set for small sets.
j. calculate the size of the Cartesian product of sets, and of the power set of a set.

UNIT IV: Discrete Functions and Relations

Goal. The student will demonstrate the ability to use discrete functions and relations to solve problems.

Objectives – The student will be able to:

a. use the composition of logarithms and the floor or ceiling functions to solve problems.
b. calculate values for iterated function sequences for various choices of a_0.
c. give complete or partial arrow diagrams for iterated functions.
d. use algebraic methods to locate all cycles of length 1 and length 2 for given functions.
e. use the principle of mathematical induction to prove that a given sequence eventually reaches one of several cycles.
f. apply properties of iterated functions to construct the Sierpinski triangle.

UNIT V: Combinatorics

Goal. The student will demonstrate the ability to use a problem-solving approach in applying counting techniques in order to determine probabilities.

Objectives – The student will be able to:

a. demonstrate that two sets are the same size (without knowing the size of either set) by describing a 1-1 correspondence between the two sets.
b. apply the General Addition Rule for Counting.
c. apply the Multiplication Principle of Counting.
d. apply the Complement Rule.
e. interpret and simplify expressions involving factorial notation.
f. solve counting problems involving permutations, combinations, and other systematic counting methods.
g. apply the Binomial Theorem to determine the coefficient of x^k in various expressions.
h. relate Pascal's Triangle to the Binomial Theorem.
i. use r-combinations to count binary sequences of length n with r zeros.
j. count unordered lists with repetitions
k. count non-negative integer solutions to equations such as $a + b + c = 10$.
l. find a closed formula for a recurrence relation.

UNIT VI: Probability & Games

Goal. The student will demonstrate the ability to use a problem-solving approach to determine probabilities.

Objectives – The student will be able to:

a. describe the sample space for an event
b. draw a Venn Diagram to represent the probability of an event
c. determine the probability of the union and intersection of two or more events
d. determine the probability of the complement of an event
e. determine whether two events are mutually exclusive (disjoint)
f. determine whether two events are independent
g. calculate conditional probabilities
h. apply Bayes' Rule using a tree diagram to calculate reverse conditional probabilities
i. use transition matrices to represent Markov chains
j. determine whether a matrix is a regular transition matrix
k. find the stable distribution of a regular transition matrix

UNIT VII: Cryptological Mathematics

Goal. The student will demonstrate the ability to use a problem-solving approach to encipher and decipher messages using various mathematical strategies including number theory, modular arithmetic, combinatorics, probability, and matrices.

Objectives – The student will be able to:

a. Encipher and decipher a message using monoalphabetic substitution ciphers including additive, multiplicative, and affine ciphers
b. Encipher and decipher a message using the Vigenère Square
c. Compute the Index of Coincidence of a message and use the Kasiski Test to approximate the length of the keyword
d. Encipher and decipher messages using Playfair's system
e. Encipher and decipher messages using Hill's system

UNIT VIII: Graphs and Circuits

Goal. The student will demonstrate the ability to use a problem-solving approach to model practical problems using graphs and circuits.

Objectives – The student will be able to:

a. determine whether two graphs are isomorphic
b. determine whether a graph is planar or nonplanar
c. find subgraphs, supergraphs, and expansions of graphs
d. describe cyclic graphs, null graphs, and complete graphs with *v* vertices
e. determine the valence (degree) and connectedness of a graph
f. use graphs to model situations in which the vertices represent objects, and edges represent a particular relationship between objects
g. represent the vertices and edges of a graph as an adjacency matrix, and use the matrix to solve problems
h. determine whether a graph has an Euler Circuit or Path, and if so, find it
i. determine whether a graph has a Hamilton Circuit or Path, and if so, find it
j. determine the number of Hamilton Circuits for a complete graph with *n* vertices