Linear Algebra GT Essential Curriculum

The Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

The Mathematical Content Standards

The Mathematical Content Standards (Essential Curriculum) that follow are designed to promote a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the mathematical practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. *In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.* In this respect, those content standards that set an expectation of understanding are potential "points of intersection" between the Mathematical Content Standards and the Mathematical Practices.

Unit 1: Linear Systems

Part 1: Linear Equations
LA.LS.A: Identify, construct and evaluate systems of linear equations.
LA.LS.A.1: Determine whether a linear system has no solution, a unique solution or many solutions.
LA.LS.A.2: Explain graphically what it means for a linear system of 2 or 3 variables to be consistent.
LA.LS.A.3: Represent a linear system with an augmented matrix.

Part 2: Elimination
LA.LS.B: Elimination reveals important details about a linear system.
LA.LS.B.1: Use elimination on an augmented matrix to find the solution set of a linear

system of equations.
LA.LS.B.2: Understand elimination establishes both existence and uniqueness of a solu- tion set to a particular linear system
LA.LS.B.3: Find particular solutions of a linear system of equations by making particular choices for the values of the free variables.
LA.LS.B.4: Transform an augmented matrix into echelon form.
LA.LS.B.5: Use the echelon form of an augmented matrix to determine the existence and uniqueness of the solution set of the linear system.

Part 3: Vector Equations
LA.LS.C: A system of linear equations can be expressed as a vector equation.
LA.LS.C.1: Interpret a linear system of equations as a vector equation and vice versa.
LA.LS.C.2: Identify, construct and compute with column vectors.
LA.LS.C.3: Interpret geometrically column vectors with 2 or 3 entries.
LA.LS.C.4: Identify, construct and interpret a linear system as a linear combinations of vectors in a vector equation.

Part 4: Matrix Equations
LA.LS.D: A system of linear equations can be expressed as a matrix equation.
LA.LS.D.1: Transform a vector equation into a matrix equation and vice versa.
LA.LS.D.2: Compute the product of an $m \times n$ matrix A and an $n \times 1$ column vector x.
LA.LS.D.3: An augmented matrix can be used to solve both vector and matrix equations.
LA.LS.D.4: Determine if the matrix equation $Ax = b$ is consistent by determining if b is in the span of the columns of A. Define the transpose of a matrix A denoted AT , as well as use the properties of transposes in evaluating matrix expressions.

Unit 2: Matrix Algebra

Part 1: Matrix Operations
LA.MA.A: Matrices facilitate the study of systems of linear equations.
LA.MA.A.1: Define and note a matrix and its entries.
LA.MA.A.2: Evaluate or compute matrix expressions involving matrix multiplication, matrix addition or multiplication by a scalar.
LA.MA.A.3: Interpret matrix multiplication as the composition of linear transformations.
LA.MA.A.4: Interpret each column of the matrix product AB as the linear combination of the columns of A using weights rom the appropriate column of B.

Part 2. The Inverse of a Matrix
LA.MA.B: Only square matrices are invertible, and permits the connection between important concepts of square linear systems of equations.
LA.MA.B.1: Define the inverse of a matrix A as the existence of a matrix C such that $CA = AC = I$.
LA.MA.B.2: A matrix equation $Ax = b$ has a unique solution if A is invertible.
LA.MA.B.3: Identify, construct and use elementary matrices.
LA.MA.B.4: A matrix A is invertible if and only if it can be written as a product of elementary matrices.

Part 3: Subspaces
LA.MA.C: Subspaces are important sets of vectors in Rn and provide important information about the matrix equation Ax = b.
LA.MA.C.1: Identify and use the 3 properties that define a subspace.
LA.MA.C.2:

Part 4: Determinants
LA.MA.D: The determinant of a matrix provides a variety of important practical and theoretical information.
LA.MA.D.1: Determine whether a 2×2 matrix is invertible.
LA.MA.D.2: Compute the determinant of an arbitrary square matrix by means of cofactors.
LA.MA.D.3: Use the determinant of a square matrix A in order to determine if A is invertible.
LA.MA.D.4: Use Cramer's Rule to solve linear systems of equations.
LA.MA.D.5: Interpret the determinant as of a measure of area or volume for a 2×2 or 3×3 matrix, respectively. D.

Unit 3: Vector Spaces

Part 1: Vector Spaces & Subspaces
LA.VS.A: Vector spaces and subspaces provide an alternative point of view from which to study different mathematical concepts like polynomials.
LA.VS.A.1: Determine whether particular sets are vector spaces by using the definition and axioms for a vector space.
Determine whether particular sets are subspaces by using the definition of a vector sub- space.

Part 2: Special types of Subspaces.
LA.SS.B: Subspaces can be interpreted as the set of all solutions to a homogeneous system or as the linear combinations of an indexed set of vectors.
LA.SS.B.1: Understand the set of all solutions to the homogeneous matrix equation Ax = 0 is the null space, and is a subspace of Rn.
LA.SS.B.2: Since there is no obvious relation between the matrix A and the vectors contained in Nul A, solving Ax = 0 generates an explicit description of the vectors in the null space of A.
LA.SS.B.3: Understand the set of all linear combinations of the columns of a matrix A is the column space and is a subspace of Rn
LA.SS.B.4: The column space of a matrix A can be interpreted as the range of some linear transformation.

Part 3: Linearly Independent Sets & Bases
LA.IS.C: A basis is a spanning set that is both as small and as large as possible.
LA.IS.C.1: Construct a basis from a spanning set by deleting unnecessary vectors.
LA.IS.C.2: A basis for Nul A has the same number of vectors as free variables when the null space of A contains nonzero vectors.
LA.IS.C.3: A basis for Col A is determined by examining the pivots columns of the RREF of A. Namely, the columns of A and the RREF of A have the same linear dependence relationships.
LA.IS.C.4: A basis for a vector space V can be used map a vector x in V to a vector in R^n.

LA.IS.C.5: If a vector space V has n basis vectors then any set in V with more than n vectors is linearly dependent.
LA.IS.C.6: The dimension of a subspace is the number of basis vectors for the subspace.

Part 4: Rank

LA.R.D: The rank of a matrix A is the dimension of the column space of A, and contains important information about the invertibility of a square matrix.
LA.R.D.1: The rank of A plus the dimension of the null space of A is equal to the number of columns of A.
LA.R.D.2; The rank of A is equal to the number of pivot columns of A and the dimension of the null space of A is equal to the number of non-pivot columns of A.