The Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

The Mathematical Content Standards

The Mathematical Content Standards (Essential Curriculum) that follow are designed to promote a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the mathematical practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards that set an expectation of understanding are potential “points of intersection” between the Mathematical Content Standards and the Mathematical Practices.

Instructional Note: The following standards should be emphasized throughout all units of instruction. These standards should not be taught in isolation.

Experiment with transformations in the plane.
G.CO.A.1 Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

Apply geometric concepts in modeling situations.
G.MG.A.1 Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder)
Apply geometric concepts in modeling situations.
G.MG.A.2 Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).

Apply geometric concepts in modeling situations.
G.MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

**Unit 1: Transformations, Similarity, and Congruence**

**Part I: Transformations and the Coordinate Plane**

Note: Students taking Geometry in Grade 8 and Grade 10 have had instruction on transformations, Pythagorean theorem, and distance formula in Pre-Algebra. See standards 8.G.A.1-8.G.A.5 and 8.G.B.6-8.G.B.8. Students in taking Geometry in Grade 9 may need pre-teaching with these standards. Check for understanding and provide support as needed.

**G.GPE.B Use coordinates to prove simple geometric theorems algebraically.**
G.GPE.B.4 Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle.

G.GPE.B.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

G.GPE.B.5 Prove the slope criteria for parallel and perpendicular lines and uses them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

**G.CO.D Make geometric constructions.**
G.CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Constructions include: constructing perpendicular lines, constructing a line parallel to a given line through a point not on the line.

**G.CO.A Experiment with transformations in the plane.**
G.CO.A.2 Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

G.CO.A.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
G.CO.C Prove geometric theorems.
G.CO.C.9 Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent.*

**Part II: Similarity**

G.SRT.A Understand similarity in terms of similarity transformations.
G.SRT.A.1 Verify experimentally the properties of dilations given by a center and a scale factor.
   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

G.SRT.A.2 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

**Part III: Congruence**

G.CO.A Experiment with transformations in the plane.
G.CO.A.3 Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

G.CO.C Prove geometric theorems.
G.CO.C.11 Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.* *(Note: This standard will be revisited in Unit 2. In this unit, focus on verifying relationships in the coordinate plane.)*

G.CO.B Understand congruence in terms of rigid motions.
G.CO.B.6 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

G.CO.A Experiment with transformations in the plane.
G.CO.A.5 Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

G.CO.D Make geometric constructions.
G.CO.D.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric
Prove geometric theorems. (This standard will be embedded throughout this unit)

G.CO.C.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to

G.SRT.A Understand similarity in terms of similarity transformations.
G.SRT.A.3 Use properties of similarity transformations to establish the AA criterion for two triangles to be similar.

G.SRT.B Prove theorems involving similarity.
G.SRT.B.5 Use similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

G.SRT.B.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

Prove geometric theorems. (This standard will be embedded throughout this unit)

G.CO.C.10 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to

Prove theorems involving similarity.
G.SRT.B.4 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

Use coordinates to prove simple geometric theorems algebraically.
G.GPE.B.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

Prove geometric theorems. (This standard will be embedded throughout this unit)
G.CO.C.10 Prove theorems about triangles. Theorems include: the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length

Make geometric constructions.
G.CO.C.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment.
G.CO.C Prove geometric theorems.  (This standard will be embedded throughout this unit)
G.CO.C.10 Prove theorems about triangles.  Theorems include: the medians of a triangle meet at a point.

G.CO.B Understand congruence in terms of rigid motions.
G.CO.B.7 Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

G.CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove theorems involving similarity.
G.SRT.B.5 Use congruence criteria for triangles to solve problems and to prove relationships in geometric figures.

Prove geometric theorems.
G.CO.C.9 Prove theorems about lines and angles. Theorems include: points on a perpendicular bisector of a line segment are exactly those equidistant from the segment’s endpoints.

G.CO.C Prove geometric theorems.  (This standard will be embedded throughout this unit)
G.CO.C.10 Prove theorems about triangles.  Theorems include: base angles of an isosceles triangle are congruent

Prove theorems involving similarity.
G.SRT.B.5 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Prove geometric theorems.
G.CO.C.11 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.  (Note: Revisit this Unit 1 standard.  Use triangle theorems and postulates to prove these theorems.)

Part II: Right Triangle Trigonometry

G.SRT.B Prove theorems involving similarity.
G.SRT.B.4 Prove theorems about triangles.  Theorems include: the Pythagorean Theorem proved using triangle similarity.
G.SRT.B.6 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

G.SRT.C Define trigonometric ratios and solve problems involving right triangles. 
G.SRT.C.7 Explain and use the relationship between the sine and cosine of complementary angles.

G.SRT.C.8 Use trigonometric ratios and the Pythagorean Theorem to solve right triangle in applied problems.

G.SRT.D Apply trigonometry to general triangles. (Geometry GT) 
G.SRT.D.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems. (Geometry GT)

G.SRT.D.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). (Geometry GT)

G.SRT.D.9(+) Derive the formula \( A = \frac{1}{2} ab \sin(C) \) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. (Geometry GT)

Unit 3: Circles, Proof, and Constructions

G.C.A Understand and apply theorems about circles. 
G.C.A.1 Prove that all circles are similar.

G.C.A.2 Identify and describe relationships among inscribed angles, radii, and chords. 
\textit{Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.}

G.C.A.4 (+) Construct a tangent line from a point outside a given circle to the circle. (Geometry GT)

G.C.A.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

G.CO.C Make geometric constructions. 
G.CO.C.13 Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

G.GMD.A Explain volume formulas and use them to solve problems. 
G.GMD.A.1 Give an informal argument for the formulas for the circumference of a circle, area of a circle. \textit{Use dissection arguments and informal limit arguments.}
G.C.B Find arc lengths of sectors of circles.
G.C.B.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

G.GPE.A Translate between the geometric description and the equation for a conic section.
G.GPE.A.1 Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

G.GPE.B Use coordinates to prove simple geometric theorems algebraically.
G.GPE.B.4 Use coordinates to prove simple geometric theorems algebraically. i.e. prove or disprove that the point lies on the circle centered at the origin and containing the point (0, 2).

Unit 4: Extending to Three Dimensions

G.GMD.B Visualize the relation between two-dimensional and three-dimensional objects.
G.GMD.B.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

G.GMD.A Explain volume formulas and use them to solve problems.
G.GMD.A.1 Give an informal argument for the formulas for the volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri’s principle, and informal limit arguments.

G.GMD.A.3 Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

G.MG.A Apply geometric concepts in modeling situations.
G.MG.A.3 Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

Unit 5: Applications of Probability

Part I: Probability of Compound Events

S.CP.A Understand independence and conditional probability and use them to interpret data.
S.CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or”, “and”, “not”).

S.CP.B Use the rules of probability to compute probabilities of compound events in a uniform probability model.
S.CP.B.7 Apply the Addition Rule, \( P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \), and interpret the answer in terms of the model.

S.CP.A Understand independence and conditional probability and use them to interpret data.
S.CP.A.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S.CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.

Part II: Conditional Probability

S.CP.A Understand independence and conditional probability and use them to interpret data.
S.CP.A.3 Understand the conditional probability of A given B as \( P(A \text{ and } B)/P(B) \), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

S.CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

S.CP.B Use the rules of probability to compute probabilities of compound events in a uniform probability model.
S.CP.B.6 Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model.

S.CP.B.8 (+) Apply the general Multiplication Rule in a uniform probability model, \( P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B) \), and interpret the answer in terms of the model. (Geometry GT)
S.CP.B.9 (+) Use permutations and combinations to compute probabilities of compound events and solve problems. *(Geometry GT)*

**S.MD.B Use probability to evaluate outcomes of decisions.**
S.MD.B.6 (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). *(Geometry GT)*

S.MD.B.7 (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulled a hockey goalie at the end of a game). *(Geometry GT)*