The Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

The Mathematical Content Standards

The Mathematical Content Standards (Essential Curriculum) that follow are designed to promote a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the mathematical practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards that set an expectation of understanding are potential “points of intersection” between the Mathematical Content Standards and the Mathematical Practices.

Instructional Note: The following standards should be emphasized throughout all units of instruction. These standards should not be taught in isolation.

N.Q.A Reason quantitatively and use units to solve problems.
N.Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.

A.SSE.A Interpret the structure of expressions.
A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.

A.CED.A Create equations that describe numbers or relationships.
A.CED.A.1 Create equations and inequalities in one variable and use them to solve problems.

F.IF.B Interpret functions that arise in applications in terms of a context.
F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.
F.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

S.ID.B Summarize, represent, and interpret data on quantitative variables.
S.ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, exponential, and quadratic models.
   b. Informally assess the fit of a function by plotting and analyzing residuals.

Unit 1: Introduction to Families of Functions

F.IF.B. Interpret functions that arise in applications in terms of a context.
F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*. (Note: Emphasize the selection of a model based on the context and behavior of the data.)

S.ID.B Summarize, represent, and interpret data on quantitative variables.
S.ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.
   b. Informally assess the fit of a linear function by plotting and analyzing residuals.
   c. Fit a linear function for a scatter plot that suggests a linear association.

F.IF.C Analyze functions using different representations.  F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Unit 2: Expanding Understanding of Quadratic Functions

F.IF.C Analyze functions using different representations  F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
N.CN.A Perform arithmetic operations with complex numbers

N.CN.A.1 Know there is a complex number \( i \) such that \( i^2 = -1 \), and every complex number has the form \( a + bi \) with \( a \) and \( b \) real.

N.CN.A.2 Use the relation \( i^2 = -1 \) and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

A.REI.B Solve equations and inequalities in one variable.
A.REI.B.4 Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in \( x \) into an equation of the form \( (x – p)^2 = q \) that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for \( x^2 = 49 \)), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as \( a + bi \) for real numbers \( a \) and \( b \).

N.CN.C Use complex numbers in polynomial identities and equations.
N.CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.

A.REI.C Solve Systems of Equations
A.REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

N.CN.C Use complex numbers in polynomial identities and equations.
N.CN.C.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. (Algebra II GT only)

N.CN.C.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite \( x^2 + 4 \) as \( (x + 2i)(x – 2i) \). (Algebra II GT only)

G.GPE.A Use coordinates to prove simple geometric theorems algebraically. G.GPE.A.2 Derive the equation of a parabola given a focus and directrix. (Algebra II GT only)

Unit 3: Polynomial Functions

A.APR.A Perform arithmetic operations on polynomials.
A.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

F.BF.A Build a function that models a relationship between two quantities.
F.BF.A.1 Write a function that describes a relationship between two quantities.
b. Combine standard function types using arithmetic operations.

**Analyze functions using different representations.**

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

**F.IF.B Interpret functions that arise in applications in terms of a context.**

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior.*

**F.BF.B Build new functions from existing functions.**

F.BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

**F.IF.C Analyze functions using different representations.**

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

**A.APR.B Understand the relationship between zeros and factors of polynomials.**

A.APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

**A.REI.D Represent and solve equations graphically.**

A.REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.

**A.APR.C Use polynomial identities to solve problems.**

A.APR.C.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

A.APR.C.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of $x$ and $y$ for a positive integer $n$, where $x$ and $y$ are any numbers, with coefficients determined for example by Pascal’s triangle. (Algebra II GT only)
**Unit 4: Radical and Rational Functions**

**F.IF.C Analyze functions using different representations.**
F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

**N.RN.A Extend the properties of exponents to rational exponents.**
N.RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

N.RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

**Understand solving equations as a process of reasoning and explain the reasoning.**
A.REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

**F.IF.C Analyze functions using different representations.**
F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

**A.APR.B Understand the relationship between zeros and factors of polynomials.**
A.APR.B.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number $a$, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

**A.APR.D Rewrite rational expressions.**
A.APR.D.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

**A.REI.D Represent and solve equations graphically.**
A.REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, and rational functions.*
Unit 5: Exponential and Logarithmic Functions

F.IF.A Understand the concept of a function and use function notation.
F.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

F.BF.A Build a function that models a relationship between two quantities.
F.BF.A.2 Write geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.LE.A Construct and compare linear and exponential models and solve problems.
F.LE.A.2 Construct exponential functions, including geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.B Interpret expressions for functions in terms of the situations they model.
F.LE.B.5 Interpret the parameters in an exponential function in terms of a context.

A.SSE.B Write expressions in equivalent forms to solve problems.
A.SSE.B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

F.BF.B Build new functions from existing functions.
F.BF.B.4 Find inverse functions. Focus on linear functions, but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, $x > 0$.

\begin{itemize}
  \item[a.] Solve an equation of the form $f(x) = c$ for a simple function $f$ that has an inverse and write an expression for the inverse.
\end{itemize}

F.IF.C Analyze functions using different representations.
F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

\begin{itemize}
  \item[e.] Graph exponential and logarithmic functions, showing intercepts and end behavior.
\end{itemize}

F.IF.B Interpret linear and exponential functions that arise in applications in terms of a context.
F.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.IF.C Analyze functions using different representations.
F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

F.BF.A Build a function that models a relationship between two quantities.
F.BF.A.1 Write a function that describes a relationship between two quantities.*
b. Combine standard function types using arithmetic operations.

**A.SSE.B Write expressions in equivalent forms to solve problems**

A.SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

c. Use the properties of exponents to transform expressions for exponential functions.

**F.BF.B Build new functions from existing functions.**

F.BF.B.3 Identify the effect on the graph of replacing \( f(x) \) by \( f(x) + k \), \( k f(x) \), \( f(kx) \), and \( f(x + k) \) for specific values of \( k \) (both positive and negative); find the value of \( k \) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

**F.REI.D Represent and solve equations graphically.**

F.REI.D.11 Explain why the \( x \)-coordinates of the points where the graphs of the equations \( y = f(x) \) and \( y = g(x) \) intersect are the solutions of the equation \( f(x) = g(x) \); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where \( f(x) \) and/or \( g(x) \) are linear, exponential, and logarithmic functions.*

**F.LE.A Construct and compare linear, quadratic, and exponential models and solve problems.**

F.LE.A.4 For exponential models, express as a logarithm the solution to \( a \ b^{ct} = d \), where \( a \), \( c \), and \( d \) are numbers and the base \( b \) is 2, 10, or \( e \); evaluate the logarithm using technology.

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**Unit 6: Trigonometric Functions**

**F.TF.A Extend the domain of trigonometric identities.**

F.TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

**F.IF.C Analyze functions using different representations.**

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

e. Graph trigonometric functions, showing period, midline, and amplitude.

**F.IF.B Interpret functions that arise in applications in terms of a context.**

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. **Key features include:** intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums;
symmetries; end behavior; and periodicity.*

**F.TF.B Model periodic phenomena with trigonometric functions.**
F.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*

**F.TF.C Prove and apply trigonometric identities.**
F.TF.C.8 Prove the Pythagorean identity \( \sin^2(\theta) + \cos^2(\theta) = 1 \) and use it to find \( \sin(\theta) \), \( \cos(\theta) \), or \( \tan(\theta) \), given \( \sin(\theta) \), \( \cos(\theta) \), or \( \tan(\theta) \), and the quadrant of the angle.

**Unit 7: Inferences and Conclusions from Data**

**S.ID.A Summarize, represent, and interpret data on a single count or measurement variable.**
S.ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

**S.IC.A Understand and evaluate random processes underlying statistical experiments.**
S.IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

**Make inferences and justify conclusions from sample surveys, experiments, and observational studies.**
S.IC.B.3 Recognize the purposes of and differences among sample surveys; explain how randomization relates to each.

**S.IC.A Understand and evaluate random processes underlying statistical experiments.**
S.IC.A.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

**Make inferences and justify conclusions from sample surveys, experiments, and observational studies.**
S.IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

S.IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

S.IC.B.6 Evaluate reports based on data.
Additional Standards Assessed on PARCC:

The following standards were taught in previous grades. Check for understanding and review as needed.

Solve system of equations

A.REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. (3x3 system) \textbf{Algebra I will be expanded next year--may need in transition year})

Geometry Unit: Applications of Probability Understand independence and conditional probability and use them to interpret data.

S.CP.A.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or”, “and”, “not”).

S.CP.A.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

S.CP.A.3 Understand the conditional probability of A given B as \(P(A \text{ and } B)/P(B)\), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.

S.CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.

S.CP.A.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations.