The Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

The Mathematical Content Standards

The Mathematical Content Standards (Essential Curriculum) that follow are designed to promote a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the mathematical practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices. In this respect, those content standards that set an expectation of understanding are potential “points of intersection” between the Mathematical Content Standards and the Mathematical Practices.

Overarching Standards

Instructional Note: The following standards should be emphasized throughout all units of instruction. These standards should not be taught in isolation.

Reason quantitatively and use units to solve problems.
N.Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N.Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.

N.Q.A.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
Interpret the structure of expressions.
A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context. (Note: Limit to linear expressions and to exponential expressions with integer exponents.)
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

Create equations that describe numbers or relationships.
A.CED.A.1 Create equations and inequalities in one variable and use them to solve problems.

Understand solving equations as a process of reasoning and explain the reasoning.
A.REI.A.1 Explain each step in solving a linear equation as following from the equity of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

Unit 1: Representing Functional Relationships

Understand the concept of a function and use function notation.
F.IF.A.1 Understanding that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \).

F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Create equations that describe numbers or relationships.
A.CED.A.2 Create linear equations to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Represent and solve equations and inequalities graphically.
A.REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

Unit 2: Linear and Exponential Relationships

Part I: Representing Linear and Exponential Functions

Construct and compare linear and exponential models and solve problems.
F.LE.A.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals; and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity change at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

**Understand the concept of a function and use function notation.**

F.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *Note: This standard should serve as an introduction to arithmetic and geometric sequences to lead to linear and exponential functions.*

**Build a function that models a relationship between two quantities.**

F.BF.A.1 Write a function that describes a relationship between two quantities.
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F.BF.A.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

**Construct and compare linear and exponential models and solve problems.**

F.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.A.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasingly linearly or exponentially.

**Analyze linear and exponential functions using different representations.**

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   a. Graph linear and exponential functions and show intercepts, maxima, and minima.
   e. Graph exponential functions, showing intercepts and end behavior.

**Solve equations and inequalities in one variable.**

A.REI.B.3 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. *Note: Students have solved multi-step linear equations in Grade 8. Pretest this standard and review as needed.*

**Create equations that describe numbers or relationships.**

A.CED.A.4 Rearrange linear formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

**Interpret linear and exponential functions that arise in applications in terms of a context.**

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts, intervals where the function is increasing, decreasing, positive, or negative, and end behavior.*
F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

**Build new functions from existing functions.**
F.BF.B.3 Identify the effect of the graph of replacing \( f(x) \) by \( f(x) + k \), \( k f(x) \), \( f(kx) \), and \( f(x + k) \) for specific values of \( k \) (both positive and negative); find the values of \( k \) given the graphs. Experiment with cases that illustrate an explanation of the effects on the graph using technology. *Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.*

**Interpret expressions for functions in terms of the situations they model.**
F.LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context. *Limit exponential functions to those of the form \( f(x) = b^x + k \).*

**Analyze linear and exponential functions using different representations.**
F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

**Part II: Modeling Data with Linear and Exponential Functions**

*Note: Students currently in high school were taught standards 8.SP.A.1-3 last year.*
*Check for understanding and review as needed. Students currently in Grades 7 and 8 need to be introduced to standards 8.SP.A.1-2.*

**Interpret linear models.**
S.ID.C.9 Distinguish between correlation and causation.

**Investigate patterns of association in bivariate data.**
8.SP.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. *(Grades 7 & 8 only)*

8.SP.A.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line. *(Grades 7 & 8 only)*

**Summarize, represent, and interpret data on quantitative variables.**
S.ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
  a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.*
b. Informally assess the fit of a linear function by plotting and analyzing residuals.
c. Fit a linear function for a scatter plot that suggests a linear association.

**Interpret linear models.**
S.ID.C.8 Compute (using technology) and interpret the correlation coefficient of a linear fit.

S.ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

**Part III: Systems of Equations and Inequalities**

*Note:* Students have learned to solve systems of equations in Grade 8. Check for understanding and review A.REI.5, A.REI.6, and A.REI.11 as needed. Students have not yet explored parallel and perpendicular lines. Be sure to include these special cases.

**Solve systems of equations.**
A.REI.C.5 Prove that, given a system of equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

A.REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. *Note: Extend this standard to include three equations and three unknowns.*

**Represent and solve equations and inequalities graphically.**
A.REI.D.11 Explain why the $x$-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear and exponential functions.

**Create equations that describe numbers or relationships.**
A.CED.A.3 Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

**Represent and solve equations and inequalities graphically.**
A.REI.D.12 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

**Unit 3: Quadratic Functions and Modeling**

**Part I: Graphical Analysis and Modeling of Quadratic Functions**
Perform arithmetic operations on polynomials.
A.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Focus on polynomial expressions that are linear or quadratic in a positive integer power of \( x \).

Build a function that models a relationship between two quantities.
F.BF.A.1 Write a quadratic function that describes a relationship between two quantities.
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   b. Combine standard function types using arithmetic operations.

Analyze functions using different representations.
F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Interpret functions that arise in applications in terms of a context.
F.IF.B.4 For a quadratic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts, intervals where the function is increasing, decreasing, positive, or negative, relative maximums and minimums, symmetries, and end behavior.

F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationships it describes.

Build new functions from existing functions.
F.BF.B.3 Identify the effect on the graph of replacing \( f(x) \) by \( f(x) + k, k f(x), f(kx), \) and \( f(x + k) \) for specific values of \( k \) (both positive and negative); find the value of \( k \) given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Summarize, represent, and interpret data on quantitative variables.
S.ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data.
   b. Informally assess the fit of a linear function by plotting and analyzing residuals.

Part II: Algebraic Analysis of Quadratic Functions

Interpret the structure of expressions.
A.SSE.A.1 Interpret quadratic expressions that represent a quantity in terms of its context.
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.

**Write expressions in equivalent forms to solve problems.**
A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

**Analyze functions using different representations.**
F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- Use the process of factoring and completing the square in a quadratic function to show zeroes, extreme values, and symmetry of the graph, and interpret these in terms of a context.

**Write expressions in equivalent forms to solve problems.**
A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- Factor a quadratic expression to reveal the zeros of the function it defines.

**Solve equations and inequalities in one variable.**

*Instructional Note: Students should learn of the existence of the complex number system, but will not solve quadratics with complex solutions until Algebra II.*

A.REI.B.4 Solve quadratic equations in one variable.

- Solve quadratic equations by inspection (e.g., for \(x^2 = 49\)), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as \(a \pm bi\) for real numbers \(a\) and \(b\).

**Represent and solve equations and inequalities graphically.**
A.REI.D.11 Explain why the \(x\)-coordinates of the points where the graphs of the equations \(y = f(x)\) and \(y = g(x)\) intersect are the solutions of the equation \(f(x) = g(x)\); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where \(f(x)\) and/or \(g(x)\) are linear and exponential functions.

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**Unit 4: Descriptive Statistics**

**Summarize, represent, and interpret data on a single count or measurement variable.**
S.ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots). *Note: This will be the students’ first exposure to histograms.*
S.ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different sets.

S.ID.A.3 Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

**Summarize, represent, and interpret data on two categorical and quantitative variables.**

S.ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

*Note: Students currently in high school have been introduced to two-way tables last year. Students currently in Grades 7 & 8 have not been introduced to two-way tables.*

**Unit 5: Modeling with Other Functions**

**Analyze functions using different representations.**

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

**Extend the properties of exponents to rational exponents.**

N.RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold so $(5^{1/3})^3$ must equal 5.*

N.RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.