Differential Equations GT
Essential Curriculum

UNIT 1: Elementary Methods

Goal. The student will demonstrate the ability to use a problem-solving approach to solve elementary differential equations. The student will see explicit proofs to establish a firm foundation for subsequent mathematical truth.

Objectives - The student will be able to:

a. Explore analytically and qualitatively the two math models, y' - 1/x and a falling object equation.
b. Understand basic vocabulary and terminology.
c. Define and used Linear Operators.
d. State and use The Uniqueness Theorem.
e. Solve and apply y'=ky to growth and decay.
f. Solve Variables Separable equations (including constant solutions).
g. Use the total differential to generate families of orthogonal trajectories.
h. Apply Kirchoff's Law to electric circuits.
i. Generate direction fields using tangent segments, isoclines and a graphing calculator.
j. Solve first order linear equations.
k. Use linear first order theory to derive the relationship I-->V/R for an RL closed circuit.
l. Solve exact differential equations.
m. Prove and apply the test for exactness.
n. Use integration factor theory to make equations exact.
o. Match given slope fields with given differential equations.
p. Solve an application problem involving the velocity of a falling object, numerically, analytically and qualitatively (slope field).
q. Solve equations having the form y' = a(t,y)/b(t,y) where a(t,y) and b(t,y) are both homogeneous of degrees.

UNIT 2: Constant Coefficient Equations

Goal. The student will demonstrate the ability to use a problem-solving approach to solve constant coefficient equations using numerical and graphical analyses.

Objectives - The student will be able to:

a. Solve Constant coefficient equations.
b. Use Euler Equivalence to write solutions in harmonic form.
c. Define a vector space.
d. Apply the concepts of linear independence and linear dependence for scalar quantities and functions of dimension one.
e. Apply the concepts of linear independence and linear dependence for scalar quantities and functions of dimension one.
f. Obtain fundamental sets of solutions.
g. Define the Wronskian and explain the relationship between the Wronskian and the linear independence of differential equations
h. Obtain fundamental sets of solutions of second order, constant coefficient, and homogeneous differential equations.

i. Solve applications using harmonic springs, including the undamped, underdamped and overdamped cases.
j. Drawn and analyze phase plane portraits for all the cases of the harmonic spring.
k. Determine the direction of motion for equations of closed orbits.
l. Solve the underdamped case, and put the solutions in phase amplitude form and sketch the solution.
m. Solve equations using reduction of order.
n. Approximate numerical solutions using both Euler's Method and the Improved Euler's Method.
o. Analyze the math model $x' = -ay$; $y' = -bx$ qualitatively and analytically.
p. Analyze the math model $x' = ay$; $y' = -bx$ qualitatively and analytically.

UNIT 3: Nonhomogeneous Methods

Goal. The student will demonstrate the ability to use a problem-solving approach to solve nonhomogeneous differential equations.

Objectives - The student will be able to:
a. Solve nonhomogeneous equations using the variation of parameters method.
b. Solve Euler equations.
c. Use the method of undetermined coefficients to solve nonhomogeneous equations .
d. Derive Abel's Formula using reduction of order methods.
e. Solve differential equations using reduction of order methods.
f. Explore an application using Archimedes' Principle.

UNIT 4: Laplace Transform

Goal. The student will demonstrate the ability to use a problem-solving approach to solve differential equations using the Laplace Transform.

Objectives - The student will be able to:
a. Define the Laplace Transform.
b. Explicitly derive the Laplace Transform of various functions.
c. Explain the necessary conditions for the existence of the Laplace Transform.
d. Prove that $L(tf(t))=-d(L(tf(t))/ds$.
e. Solve differential equations using the Laplace Transform.
f. Find Inverse Laplace Transforms using partial fractions and completing the square.
g. Solve more difficult differential equations using Laplace Transform tables and the method of Laplace.
h. Rewrite piecewise defined functions using the Heaviside Function.
i. Derive the Laplace of the Heaviside Function.
j. Explore the Dirac-Delta Function.
i. Solve equations using the Laplace Transform and all the established Laplace truth.

UNIT 5: Power Series Solutions

Goal. The student will demonstrate the ability to use a problem-solving approach to solve variable coefficient differential equations using power series.

Objectives - The student will be able to:
a. Solve variable coefficient differential equations at ordinary points using power series.
b. Determine the radius of convergence for each solution
c. Classify points as ordinary, regular singular or irregular singular.

d. Find the indicial equation for a given equation and a regular singular point.
e. Generate solution forms for regular singular points and determine the guaranteed radii of convergence
f. Solve differential equations using power series and the method of Frobenius.
g. Solve differential equations using the Taylor Series method.
h. Analyze and explore the Bessel Equation and the Gamma Function.
i. Generate Hermite and Legendre Polynomials.

UNIT 6: Elementary Systems

Goal. The student will demonstrate the ability to use a problem-solving approach to identify the linear algebra substructure for the solution of systems of differential equations. Eigenvalues and eigenvectors are introduced and used to solve linear systems.

Objectives - The student will be able to:
a Solve two-dimensional systems by elimination.
b. Write systems in matrix/vector form; obtained their solutions in vector form.
c. Change nth. order differential equations to nth. dimensional first order systems.
d. Change nth. dimensional first order systems to nth. order differential equations
e. Apply systems using closed electric circuits.
f. Use determinant properties
g. Solve linear systems of equations using the method of Gauss-Jordan.
h. Transform matrices into reduced row echelon form.
i. Find the inverse of a matrix by using elementary row operations.
j. Find the inverse of a matrix by using the adjoint matrix method.
k. Solve a system of linear equations using the inverse of the coefficient matrix.
l. Apply theorems related to eigenvalues and eigenvectors
m. Solve a linear system of first order differential equations using eigenvectors for distinct real eigenvalues.

UNIT 7: Advanced Systems

Goal. The student will demonstrate the ability to use a problem-solving approach to solve nonhomogeneous linear systems using variation of parameters and the Laplace Transform.

Objectives **-** The student will be able to:
a. Solve linear systems of differential equations using eigenvectors where the eigenvalues are not distinct.
b. Solve linear systems of differential equations using eigenvectors where the eigenvalues are complex.
c. Generate the fundamental matrix for a given linear system.
d. Solve nonhomogeneous linear systems using variation of parameters.
e. Solve nonhomogeneous linear systems using Laplace Transform.

UNIT 8: **Nonlinear Systems and Mathematical Models**

Goal. The student will demonstrate the ability to use a problem-solving approach to use qualitative analysis in determining the type of stability associated with the solutions to a nonlinear system. Mathematical models are generated and analyzed.

Objectives - The student will be able to:

a. Conduct stability analysis of a nonlinear models of pendulum motion including rough drawings of the phase plane portraits.
b. Conduct stability analysis for linear systems, including phase plane portraits.
c. Determine equilibrium points for linear and nonlinear systems.
d. Linearize nonlinear systems at their equilibrium points and determine stability using eigenvalues.
e. Categorize phase plane portraits according to their type and stability.
f. Use the Jacobian to determine the stability of nonlinear systems.
g. Draw accurate phase plane portraits, including the boundary half-lines.
h. Describe the L.F. Richardson war model and the D'Ancona/Volterra predator-prey model