

HCPSS Algebra II/Algebra II GT Essential Curriculum

The Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

The Mathematical Content Standards

The Mathematical Content Standards (Essential Curriculum) that follow are designed to promote a balanced combination of procedure and understanding. Expectations that begin with the word “understand” are often especially good opportunities to connect the mathematical practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. *In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.* In this respect, those content standards that set an expectation of understanding are potential “points of intersection” between the Mathematical Content Standards and the Mathematical Practices.

Instructional Note: The following standards should be emphasized throughout all units of instruction. These standards should not be taught in isolation.

N.Q.A Reason quantitatively and use units to solve problems.

N.Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N.Q.A.2 Define appropriate quantities for the purpose of descriptive modeling.

Unit 1: Sequences and Functions

A.SSE.B Write expressions in equivalent forms to solve problems.

A.SSE.B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

F.IF.A Understand the concept of a function and use function notation.

F.IF.A.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.

F.IF.B Interpret functions that arise in applications in terms of a context.

F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F.IF.C Analyze functions using different representations.**F.BF.A Build a function that models a relationship between two quantities.**

F.BF.A.1 Write a function that describes a relationship between two quantities.

- a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F.BF.A.2 Write geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

F.LE.A Construct and compare linear and exponential models and solve problems.

F.LE.A.2 Construct exponential functions, including geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Unit 2: Polynomials and Rational Functions**A.SSE.A Interpret the structure of expressions.**

A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

- a. Interpret parts of an expression, such as terms, factors, and coefficients.

A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.

A.SSE.B Write expressions in equivalent forms to solve problems

A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- a. Factor a quadratic expression to reveal the zeros of the function it defines.
- b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

A.SSE.B.4 Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

A.APR.A Perform arithmetic operations on polynomials.

A.APR.A.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

A.APR.B Understand the relationship between zeros and factors of polynomials.

A.APR.B.2 Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

A.APR.B.3 Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

A.APR.C Use polynomial identities to solve problems.

A.APR.C.4 Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples.

A.APR.C.5 (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n , where x and y are any numbers, with coefficients determined for example by Pascal's triangle. **(Algebra II GT only)**

A.APR.D Rewrite rational expressions.

A.APR.D.6 Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

A.CED.A Create equations that describe numbers or relationships.

A.CED.A.1 Create equations and inequalities in one variable and use them to solve problems.

A.CED.A.2 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V=IR$ to highlight resistance, R .*

A.REI.A Understand solving equations as a process of reasoning and explain the reasoning.

A.REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A.REI.C Solve system of equations

A.REI.C.6 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

A.REI.C.7 Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.

A.REI.D Represent and solve equations graphically.

A.REI.D.11 Explain why the x -coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.

F.IF.A Understand the concept of a function and use function notation.

F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.B Interpret functions that arise in applications in terms of a context.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

F.IF.B.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

F.IF.C Analyze functions using different representations.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
- d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Unit 3: Complex Numbers and Rational Exponents

N.RN.A Extend the properties of exponents to rational exponents.

N.RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

N.RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

N.CN.A Perform arithmetic operations with complex numbers

N.CN.A.1 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

N.CN.A.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N.CN.C Use complex numbers in polynomial identities and equations.

N.CN.C.7 Solve quadratic equations with real coefficients that have complex solutions.

N.CN.C.8 (+) Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$. **(Algebra II GT only)**

N.CN.C.9 (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **(Algebra II GT only)**

A.REI.A Understand solving equations as a process of reasoning and explain the reasoning.

A.REI.A.1 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A.REI.B Solve equations and inequalities in one variable.

A.REI.B.4 Solve quadratic equations in one variable.

- Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
- Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .

A.REI.D Represent and solve equations graphically.

A.REI.D.11 Explain why the x -coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.

F.IF.C Analyze functions using different representations.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- Graph square root and cube root functions.

F.BF.B Build new functions from existing functions

F.BF.B.4 Find inverse functions.

- Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = \frac{x+1}{x-1}$ for $x \neq 1$.

Unit 4: Exponential Functions and Equations

N.RN.A Extend the properties of exponents to rational exponents.

N.RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

A.SSE.A Interpret the structure of expressions.

A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context.

- b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example interpret $P(1 + r)^n$ as the product of P and a factor not depending on P .*

A.SSE.B Write expressions in equivalent forms to solve problems

A.SSE.B.3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

- c. Use the properties of exponents to transform expressions for exponential functions.

A.REI.D Represent and solve equations graphically.

A.REI.D.11 Explain why the x-coordinates of the points where the graphs of the equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations.

F.IF.A Understand the concept of a function and use function notation.

F.IF.A.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.B Interpret functions that arise in applications in terms of a context.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

F.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.

F.IF.C Analyze functions using different representations.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- e. Graph exponential and logarithmic functions, showing intercepts and end behavior.

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

- b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in function such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^{\frac{t}{10}}$, and classify them as representing exponential growth or decay.*

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

F.BF.A Build a function that models a relationship between two quantities.

F.BF.A.1 Write a function that describes a relationship between two quantities.

- a. Determine an explicit expression, a recursive process, or steps for calculation from a context.

F.LE.A Construct and compare linear, quadratic, and exponential models and solve problems.

F.LE.A.1 Distinguish between situations that can be modeled with linear function and with exponential functions.

- a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.A.2 Construct exponential functions, including geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

F.LE.A.4 For exponential models, express as a logarithm the solution to $a \cdot b^{ct} = d$, where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.

F.LE.B Interpret expressions for functions in terms of the situations they model.

F.LE.B.5 Interpret the parameters in an exponential function in terms of a context.

Unit 5: Transformations of Functions

F.IF.B Interpret linear and exponential functions that arise in applications in terms of a context.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

F.IF.C Analyze functions using different representations.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- c. Graph piecewise-defined functions, including step functions and absolute value functions.

F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

F.BF.A Build a function that models a relationship between two quantities.

F.BF.A.1 Write a function that describes a relationship between two quantities.

- b. Combine standard function types using arithmetic operations.

F.BF.B Build new functions from existing functions.

F.BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.LE.B Interpret expressions for functions in terms of the situations they model.

S.ID.B Summarize, represent, and interpret data on quantitative variables.

S.ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

- a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, exponential, and quadratic models.

Unit 6: Trigonometric Functions

N.Q.A Reason quantitatively and use units to solve problems.

N.Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

F.IF.B Interpret linear and exponential functions that arise in applications in terms of a context.

F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

F.IF.C Analyze functions using different representations.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

- e. Graph trigonometric functions, showing period, midline, and amplitude.

F.BF.B Build new functions from existing functions.

F.BF.B.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

F.TF.A Extend the domain of trigonometric identities.

F.TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F.TF.B Model periodic phenomena with trigonometric functions.

F.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F.TF.C Prove and apply trigonometric identities.

F.TF.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, and the quadrant of the angle.

Unit 7: Statistical Inferences**S.IC.A Understand and evaluate random processes underlying statistical experiments.**

S.IC.A.1 Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

S.IC.A.2 Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation.

S.IC.B Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

S.IC.B.3 Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

S.IC.B.4 Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

S.IC.B.5 Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.

S.IC.B.6 Evaluate reports based on data.

S.ID.A Summarize, represent, and interpret data on a single count or measurement variable.

S.ID.A.1 Represent data with plots on the real number line (dot plots, histograms, and box plots).

S.ID.A.2 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

S.ID.A.4 Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.