Calculus C/Multivariate Calculus – Advanced Placement G/T
Essential Curriculum

UNIT I: The Hyperbolic Functions

Goal. The student will demonstrate the ability to use a problem-solving approach to apply basic calculus concepts, including techniques for curve sketching, exponential and logarithmic functions, differentiation, and integration, to the hyperbolic functions.

Objectives – The student will be able to:

a. State the definitions of the six hyperbolic functions.
b. Sketch the graphs of the hyperbolic functions by applying such basic curve sketching techniques as asymptotes, concavity, odd or even, increasing or decreasing.
c. Identify relationships between the hyperbolic functions and recognize the similarity to the trigonometric functions.
d. Identify the derivatives of the six hyperbolic functions.
e. Identify the antiderivatives of the $y = \sinh x, y = \cosh x$ and $y = \tanh x$
f. Apply techniques of integration to solving problems involving area under and between the graphs of the hyperbolic functions.
g. Apply formulas for the inverse hyperbolic functions over the appropriate domains.

UNIT II: L'Hôpital's Rule, Improper Integrals, and Taylor Polynomials

Goal. The student will demonstrate the ability to use a problem-solving approach to find limits by applying L'Hôpital's Rule in all the appropriate indeterminate forms, and to evaluate improper integrals.

Objectives – The student will be able to:

a. Identify the indeterminate forms including $\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty, 0^{\infty}, \infty^{0}$.
b. Compute limits of functions when an indeterminate form occurs.
c. Interpret improper integrals graphically.
d. Evaluate an improper integral where one or both interval endpoints are infinite.
e. Evaluate an improper integral where the function has a discontinuity at an endpoint.
f. Evaluate an improper integral where the function has a discontinuity within the interval.
g. Write the general form of a Taylor polynomial of *n*th degree.
h. Write a Taylor (or Maclaurin) polynomial of specified degree for various functions.
i. Approximate the value of a function using a Taylor polynomial.
j. Use the Lagrange form of the remainder to identify a maximum percent error or an upper bound for error in approximation by Taylor polynomial.

UNIT III: Sequences and Series

Goal. The student will demonstrate the ability to use a problem-solving approach to apply tests for convergence and divergence of sequences and series, find the sums of convergent infinite series, and use proper sigma notation.

Objectives – The student will be able to:

a. State the definition of sequence as a function.
b. Use formulas to identify terms of a sequence.
c. State the definition of a limit of a sequence.
d. Use appropriate theorems to prove that a sequence is convergent or divergent.
e. Identify monotonic sequences.
f. State the definition of infinite series and use sigma notation to represent a series.
g. State and apply the definition of convergence of a series in terms of the sequence of partial sums.
h. Define and identify geometric series, and state conditions for convergence and divergence.
i. Compute the sum of a convergent geometric series.
j. Prove that a series converges or diverges using appropriate theorems. (e.g. *n*th term test, comparison and limit comparison test, integral test, p-series test, alternating series test, ratio test, root test.)

UNIT IV: Power Series and Taylor's Theorem

Goal. The student will demonstrate the ability to use a problem-solving approach to prove convergence or divergence of a power series and use power series, including Taylor series, to represent functions.

Objectives – The student will be able to:

a. Define a power series and represent a power series symbolically.
b. Determine the radius and interval of convergence for a power series.
c. Differentiate and integrate a power series.
d. Identify and apply power series for the function $f(x) = e^x$ and related functions.
e. Identify and apply power series for the function $f(x) = \frac{1}{1-x}$ and related functions.
f. Identify and apply power series for the functions $f(x) = \sin x, f(x) = \cos x, f(x) = \sinh x,$, and $f(x) = \cosh x$ and related functions.
g. State the definition of a Taylor series.
h. Use Taylor series to obtain power series representations for various functions.

i. Use Taylor series to approximate values of functions to a given degree of accuracy.
j. State and apply the binomial theorem.

UNIT V: Parametric Equations and Polar Coordinate System

Goal. The student will demonstrate the ability to use a problem-solving approach to define and analyze functions defined parametrically and in the polar coordinate system.

Objectives – The student will be able to:

a. Find coordinates of points on a parametrically defined function and graph the function.
b. Convert from parametric to rectangular coordinates and vice versa.
c. Apply parametric equations to analyze the cycloid graph.
d. Determine first and second derivatives of parametrically defined functions.
e. State the formula for length of curve and compute the length of curve for parametrically defined functions.
f. State the formula for area under a curve whose equations are given parametrically and compute the area under such a curve.
g. Convert polar coordinates and equations into Cartesian form and vice versa.
h. Sketch common polar graphs.
i. Test for symmetry with respect to the polar axis, the line $q = \frac{\pi}{2}$, and the pole.
j. Interpret geometrically, compute, and apply ${}^{dx}\!/_{dq}$, ${}^{dy}\!/_{dq}$, and ${}^{dr}\!/_{dq}$.
k. Convert polar equations to parametric form and apply to finding ${}^{dy}\!/_{dx}$.
l. State and apply the formula for arc length for a curve given by a polar equation.
m. State and apply the formula for area inside a curve given by a polar equation.

UNIT VI: Vectors

Goal. The student will demonstrate the ability to use a problem-solving approach to apply operations on vectors in two and three dimensions and use vectors to analyze planes, cylinders, and quadric surfaces.

Objectives – The student will be able to:

a. Perform basic operations on vectors, both geometrically and numerically.
b. Interpret points in a three-dimensional rectangular coordinate system.
c. Compute a unit vector in the direction of a given vector.
d. Express a vector as a linear combination of two other vectors.

e. Find length and midpoint of a line segment (or vector) in three dimensions.
f. Write the equation of a sphere.
g. Compute the dot product of two vectors.
h. Compute the angle between two vectors.
i. Determine whether two vectors are orthogonal.
j. Determine and apply the vector projection of one vector along another.
k. Compute the cross product of a vector in three dimensions and interpret geometrically.
l. Determine equations of lines in three dimensions in parametric and symmetric form.
m. Determine the equation of a plane in a three-dimensional coordinate system and sketch the plane.
n. Compute the distance between a point and a plane.
o. Identify and sketch the graphs of cylinders and basic types of quadric surfaces.
p. Recognize surfaces of revolution and their equations.
q. Locate points in cylindrical and spherical coordinate systems.
r. Convert coordinates and equations between cylindrical, spherical, and rectangular coordinate systems.

UNIT VII: Vector-Valued Functions

Goal. The student will demonstrate the ability to use a problem-solving approach to analyze vector-valued functions and use them in applications involving projectile motion and space curves.

Objectives – The student will be able to:
a. Sketch the graph of a vector-valued function in two dimensions.
b. Determine the Cartesian equation for a two-dimensional vector-valued function.
c. Recognize a three-dimensional vector-valued function as determining a curve in space that can be sketched as the intersection of two or more surfaces.
d. Differentiate and integrate vector-valued functions.
e. Apply vector-valued functions to solve problems involving velocity, acceleration, and projectile motion.
f. Define and compute the unit tangent and unit normal for a given vector.
g. Define and compute tangential and normal components of acceleration.
h. Define and compute arc length for a vector-valued function.
i. Define and compute curvature for a vector-valued function and for a function in Cartesian form.

UNIT VIII: Functions of Several Variables

Goal. The student will demonstrate the ability to use a problem-solving approach to interpret and analyze functions of two variables both graphically and algebraically, including using partial derivatives, directional derivatives, and gradients. The student will prepare for the Calculus BC level Advanced Placement Examination by reviewing previously taught material.

Objectives – The student will be able to:

a. Define a function of two variables.
b. Define and determine the domain and range of a function of two variables.
c. Sketch the graphs of selected functions of two variables.
d. Define and sketch level curves and contour maps.
e. Test for continuity and the existence of limits of functions of two variables.
f. Define, compute, and interpret geometrically first and second partial derivatives for functions of more than one variable.
g. Define and compute the total differential for functions of more than one variable.
h. Apply the total differential in finding relative measurement error.
i. State and apply the chain rule with one or two independent variables.
j. Define, compute, and interpret geometrically the directional derivative of a function of two variables.
k. Define, compute, and interpret geometrically the gradient of a function of two variables.
l. State and apply the relationship between the gradient and the directional derivative for a function of two variables.
m. Determine the equation of a plane tangent to a surface.
n. Identify critical points of a surface and apply to extrema of functions of two variables.
o. Apply partial derivatives to finding the equation of a least squares regression line.
p. Apply Lagrange multipliers to optimization problems.

Unit IX: Multiple Integration

Goal. The student will demonstrate the ability to use a problem-solving approach to compute and apply iterated integrals to problems involving area, volume, and center and moment of mass.

Objectives – The student will be able to:

a. Evaluate an iterated integral.
b. Represent area of a plane region using iterated integrals.
c. Apply double integrals in finding volumes of solids defined by functions of two variables.
d. Apply double integrals in finding area and volume in the polar coordinate system.
e. Apply iterated integrals in finding the mass of a lamina with constant density.
f. Define and compute moment and center of mass of a planar lamina with variable density.