Pre-Calculus Honors Essential Curriculum

The Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

The Mathematical Content Standards

The Mathematical Content Standards (Essential Curriculum) that follow are designed to promote a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the mathematical practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. *In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.* In this respect, those content standards that set an expectation of understanding are potential "points of intersection" between the Mathematical Content Standards and the Mathematical Practices.

UNIT 1: EXPANDING UNDERSTANDING OF INVERSE RELATIONSHIPS

PC.AF.A Build and interpret functions that model relationships between two quantities.
PC.AF.A.1 Identify key features of functions including domain, range, end behavior as limits to infinity and negative infinity, local (relative) and/or absolute extrema, continuity, zeros, symmetry, asymptotes, whether the function is even/odd, and average rate of change.
PC.AF.A.2 Model real-world data with functions. Justify why the model is appropriate for the problem situation.
PC.AF.A.4 Perform operations on functions and sketch the resulting graph.

MA.RSF.A Model relationships between two quantities with functions in order to solve real-world problems.

MA.RSF.A.3 Sketch and analyze the graphs of absolute value, greatest integer, and piecewise functions.
MA.RSF.A.4 Model real-world problems with special functions, identifying necessary restrictions on the model.

MA.AF.A Recognize and sketch various transformations of common functions.
MA.AF.A.1 Recognize graphs and transformations of common functions and sketch a transformed function.
F.BF.A.1c Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function time.

F.BF.B Build new functions from existing functions.
F.BF.B.4 Find inverse functions.
F.BF.B.4a Solve an equation of the form f(x)=c for a simple function f that has an inverse and write an expression for the inverse. Functions should include: linear, quadratic, exponential and their inverses.
F.BF.B.4b Verify by composition that one function is the inverse of another.
F.BF.B.4c Read values of an inverse function from a graph or a table, given that the function has an inverse.
F.BF.B.4d Produce an invertible functions from a non-invertible function by restricting the domain.

UNIT 2: TRIGONOMETRY

Part 1: Circular Functions
PC.T.A Define and evaluate circular functions.
PC.T.A.1 Define and evaluate the six trigonometric functions of an angle given a point on its terminal side. Use inverse functions to find angle measures.
PC.T.A.2 Find and state the six trigonometric functions of special and quadrantal angles.
PC.T.A.3 Find the radian measure of an angle when the circular function value is given.
PC.T.A.4 Evaluate the composition of trigonometric functions with inverses. (F.IF.A.2a)
PC.T.A.5 Solve real-world problems involving arc length, linear and angular speed, and uniform circular motion. Use co-terminal angles for problems involving multiple revolutions.
G.SRT.D.9 Derive the formula A=12ab sin(c)for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
G.SRT.D.10 Prove the Laws of Sines and Cosines and use them to solve problems.
G.SRT.D.11 Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles.

Part 2: Modeling with Graphs of Circular Functions
T.TG.A Understand the relationship between circular and trigonometric functions.
T.TG.A.1 Understand how counterclockwise motion of the radius along the unit circle translates to the cartesian graph of f(x)=sinx. Apply and extend understanding to transformations of the unit circle.

PC.T.B Use trigonometric functions to model real-world phenomena.
F.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
PC.T.B.6 Graph trigonometric parent functions represented symbolically and show key features of the graph. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, negative; maximums and minimums; symmetries; end behaviors; domain and range; amplitude, period, midline, and phase shift.
F.BF.B.3 Identify and graph transformations from the parent trigonometric functions, and identify key features.
PC.T.B.7 Write a trigonometric function that models a graph of a trigonometric function and/or describes a relationship between two quantities.
F.BF.A.1 Write a function that describes a relationship between two quantities, including more complex functions. (include composite functions)
F.BF.A.1c Compose functions.
PC.T.B.8 Use trigonometric functions to solve real-world problems. Find solutions by evaluating the function and graphing to approximate values.
PC.T.B.9 Graph the inverse trigonometric functions and identify key features of the graph. Compare the graphs the trigonometric functions and inverse trigonometric functions.

Part 3: Applying Trigonometry to Solve Equations 18-20 days
PC.T.C Prove trigonometric identities in order to solve trigonometric equations.
F.TF.C.9 Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
PC.T.C.10 Prove trigonometric identities using a variety of strategies and verify identities graphically. (Identities include Pythagorean identities and the even and odd identities.)
A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it. Simplify trigonometric expressions based on algebraic structures.
PC.T.C.11 Use sum and difference formulas, double-angle and half-angle identities to simplify, verify, and solve expressions and equations involving sine, cosine, and tangent.
PC.T.C.12 Solve trigonometric equations and inequalities algebraically and graphically, including quadratic form.
A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Produce an equivalent form of an expression that is reducible to quadratic form. (e.g. $2\cos 2x - 3\cos x + 1 = 0$)

UNIT 3: POLYNOMIAL FUNCTIONS

F.BF.A Build a functions that models a relationship between two quantities
F.BF.A.1 Write a function that describes a relationship between two quantities, including more complex functions.
F.BF.A.1c Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time

F.BF.B Build new functions from existing functions

F.BF.B.3 Identify the effect on the graph of f(x) replacing by it with f(x) + k, kf(x), f(kx), f(x+k) for specific values of (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
F.BF.B.4 Find inverse functions.
F.BF.B.4b Verify by composition that one function is the inverse of another.
F.BF.B.4c Read values of an inverse function from a graph or a table, given that the function has an inverse.
F.BF.B.4d Produce an invertible function from a non-invertible function by restricting the domain.

F.IF.A Understand the concept of a function and use function notation
F.IF.A.2a Extend evaluating functions to include operations with composite functions. For example, f(x-2) – f(x)
A.APR.C Use polynomial identities to solve problems
A.APR.C.5 Know and apply the Binomial Theorem for the expansion of (x+y)^n in powers of and x and y for a positive integer n , where and are any numbers, with coefficients determined for example by Pascal's Triangle.

F.IF.B Interpret functions that arise in application in terms of context.
F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior as limits to infinity and negative infinity; rates of change; estimating inflection points; and concavity.

F.IF.C Analyze functions using different representations
F.IF.C.7f Graph all functions including piecewise defined functions, step functions and absolute value functions
F.IF.C.7g Determine the end behavior of the graph of a polynomial function using the degree and leading coefficient
F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function
F.IF.C.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables or by a verbal description, including more complex functions).
PC.AF.A.3 Estimate limits to positive and negative infinity from graphs and tables of values.

A.SSE.A Interpret the structure of expressions
A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.

A.SSE.B Write expressions in equivalent forms to solve problems
A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

A.APR.B Understand the relationship between zeroes and factors of polynomials
A.APR.B.2 Know and apply the Remainder Theorem: For a polynomial p(x) and a number a , the remainder on division by (x-a) is p(a), so p(a)= 0 if and only if (x-a) is a factor of p(x).

N.CN.C Use Complex numbers in polynomial identities and equations
N.CN.C.8 Extend polynomial identities to the complex numbers. For example, rewrite x^2+4 as (x+2i)(x-2i).
N.CN.C.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

A.APR.D Rewrite rational expressions
A.APR.D.6 Rewrite simple rational expression in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x) a(x), b(x), q(x), and r(x) are polynomials of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.

A.CED.A Create equations that describe numbers or relationships
A.CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions.
A.CED.A.1a Create polynomial equations given real, irrational, imaginary, and/or complex roots.
A.CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.

A.REI.B Solve equations and inequalities in one variable

A.REI.C Solve systems of equations
A.REI.C.5a Solve systems of equations comprised of various combinations of all algebraic and transcendental functions in two variables.

MA.PF.A Apply understanding of polynomial functions to model and solve real-world problems.

S.ID.B Summarize, represent, and interpret data on two categorical and quantitative variables.
S.ID.B.6d Fit a function to data represented by a scatterplot; use function fitted to data.

UNIT 4: RATIONAL FUNCTIONS

N.CN.C.8 Extend polynomial identities to the complex numbers. For example, rewrite x2+ 4 as (x+2i)(x-2i).
N.CN.C.9 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

PC.AF.A Build and interpret functions that model relationships between two quantities.
PC.AF.A.1 Identify key features of functions, including domain, range, end behavior, local (relative) and/or absolute extrema, continuity, zeros, intercepts, symmetry, holes, asymptotes, and whether the function is even/odd. (Focus primarily on logarithmic, logistic, rational, step, and piecewise-defined functions.)
A.APR.D.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions.
F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing the key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
A.CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
F.IF.C.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
F.IF.C9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically, in tables or by a verbal description, including more complex functions.

PC.AF.A Build and interpret functions that model relationships between two quantities.
PC.AF.A.2 Model real-world data with functions. Justify why the model is most appropriate for the problem situation.
A.APR.D.7 (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions.
A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.
A.APR.D.6 Rewrite simple rational expressions in different forms; write a(x)b(x) in the form q(x) + r(x)b(x),where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less the degree of b(x), using inspection, long division, or for more complicated examples, a computer algebra system.
A.CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
F.BF.A.1c (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

PC.AF.A Build and interpret functions that model relationships between two quantities.
PC.AF.A.3 Calculate limits algebraically and estimate limits from graphs and tables of values.

PC.AF.B Use functions to solve equations and inequalities.
PC.AF.B.7 Solve equations and inequalities involving rational functions.

MA.RF.A Decompose a rational expression.
MA.RF.A.1 Decompose a fraction with a factorable quadratic denominator and a linear or constant numerator.
A.SSE.A.2 Use the structure of an expression to identify ways to rewrite it.
F.BF.A.1 Write a function that describes a relationship between two quantities, including more complex functions.

UNIT 5: RADICAL FUNCTIONS

N.RN.A Extend the properties of exponents to rational exponents.
N.RN.A.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.
N.RN.A.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.
A.SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
F.BF.B.3 Identify the effect on the graph of f(x) replacing f(x) by f(x) + k, kf(x), f(kx), and f(x+ k) for specific values of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
F.IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graph showing key features given a verbal description of the relationship. Key features include: domain and range; intercepts; intervals where the function is increasing, decreasing, positive, negative; relative maximums and minimums; symmetries; rate of change; end behavior with limits.

MA.RSF.A.4 Model real-world problems with radical and special functions, identifying necessary restrictions on the model.
MA.RSF.A.5 Use radical functions to model and solve real-world problems.
A.CED.A.1 Create equations and inequalities in one variable and use them to solve problems.
A.CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable.
A.REI.B Solve equations and inequalities in one variable using graphical and algebraic methods to solve radical equations and inequalities.
A.REI.A.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
A.REI.C.5a Solve systems of equations comprised of various combinations of all algebraic and transcendental functions in two functions.

UNIT 6: LOGARITHMIC AND LOGISTIC FUNCTIONS

MA.LF.A Use algebraic properties to write equivalent logarithmic equations.

A.SSE.B.3.c Use the properties of exponents to transform expressions for exponential functions. For example, the expression 1.15^t can be rewritten as (1.15^1/12)^12t = 1.012^12t to reveal the approximate monthly interest rate if the annual rate is 15%.
MA.LF.A.1 Understand the inverse relationship between exponents and logarithms, and express the inverse of an exponential function as a logarithmic function and define the natural base. Use the relationship to solve problems involving logarithms and exponents (F.BF.B.5)

F.IF.C Analyze functions using different representations.
F.IF.C.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
F.IF.C.7e. Graph logarithmic functions, showing intercepts and end behavior.
MA.LF.B.3 Sketch and analyze logarithmic functions and their transformations.

MA.LF.B Apply understanding of logarithmic functions to solve real-world problems.
A.CED.A.1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from exponential functions.
PC.AF.B.6 Use properties of logarithms, including both common and natural logarithms, to solve real-world and mathematical problems algebraically. F.LE.A.4a
PC.AF.B.6 Use the properties of logarithms, including change of base, to solve real-world and mathematical problems algebraically.
A.CED.A.3 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context.
F.LE.B.6 Build and interpret logistic functions to model real-world problems.
F.LE.B.6a Sketch and analyze logistic functions.
F.LE.B.6b Compare and contrast the exponential, logarithmic, and logistic models. Distinguish between situations that can be modeled with these functions.
F.LE.B.5a Interpret the parameters in a logistic function in terms of a context.
F.LE.B.6 Build and interpret logistic functions to model real-world problems.
F.LE.B.6c Apply understanding of logarithmic and logistic functions to solve real-world problems.

UNIT 7: EXPLORING, BUILDING, CONNECTING LIMITS

PC.AF.A Build and interpret functions that model relationships between two quantities.
PC.AF.A.3 Calculate limits algebraically and estimate limits from graphs and tables of values.

PC.AF.C Analyze and apply limits of functions to describe function behavior.
PC.AF.C.8 Use one-sided limits to describe continuity of functions.
PC.AF.C.9 Determine if a function is continuous at a point or over an interval.
CAB.GL.B.8 Describe asymptotic behavior in terms of infinite limits and limits at infinity.
F.IF.B.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
CAB.DI.A.1 Approximate rate of change from graphs and tables of values.
CAB.DI.A.2 Express the instantaneous rate of change of a function as the limit of the average rate of change.

PC.AF.C.10 Apply limits using the definition of derivative with respect to tangent lines of functions.